SCALING LAWS FOR INCIPIENT CAVITATION NOISE

by

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## I. Introduction

The noise produced by the motion of a body through a liquid differs from that produced by the motion of a body through a gas because of the possibility of cavitation in the liquid case. An adequate theory of cavitation and cavitation noise is not yet available, but the application of dimensional analysis together with the theoretical information so far obtained can yield scaling laws for this flow situation.

In section II, a brief discussion will be given of the scaling laws for hydrodynamic noise in some cases of non-cavitating flow; this discussion is included for completeness. In section III, a summary of the present information on the scaling laws for incipient cavitation noise will be presented.

## II. Hydrodynamic Noise in Non-Cavitating Flow

The first step in deriving scaling laws is to list all the parameters which are believed to affect the problem significantly. In the present paper, for completeness, a large number of parameters which may possibly influence the noise will be listed, and physical arguments will then be used to show that a number of them are probably not important in practical cases.

In non-cavitating flow, the flow field and acoustic field should be determined by the size (specified by some characteristic length L) and shape of the bodies bounding the fluid*; the fluid velocity $u_o$, pressure $p_o$, density $\rho_o$, and temperature $T_o$ at some reference point; the equation

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*An immersed body is considered to form an interior boundary.

of state, which might be written in non-dimensional form as $\rho/\rho_o =$ $f(p/p_o, T/T_o)$; the acceleration due to gravity, g; the viscosity $\mu$, thermal conductivity k, and heat capacity c of the fluid, assuming these quantities can be regarded as constant in the flow field (if not constant, their variation with temperature or other parameters must be specified). The following independent dimensionless quantities can then be formed:

(i) Dimensionless shape factors, such as width/length for each body.

(ii) Dimensionless equation of state for the fluid:

$$\rho/\rho_o = f(p/p_o, T/T_o).$$

(iii) Pressure coefficient: $\pi_o = \dfrac{p_o}{1/2\ \rho_o u_o^2}$.

(iv) "Gravitational number": $G_o = \dfrac{\rho_o g L}{p_o}$.

(v) Reynolds number: $Re_o = \dfrac{\rho_o u_o L}{\mu}$.

(vi) Kinetic/internal energy ratio: $K/I = \dfrac{1/2\ u_o^2}{cT_o}$.

(vii) Prandtl number: $Pr = \dfrac{c\mu}{k}$.

We assume that the acoustic pressure $(\Delta p)_{acoust.}$, which is a function of time in general, at any given point is completely determined by the above parameters (p, ρ, u, etc). If we define:

(viii) Dimensionless acoustic pressure: $A_o = \dfrac{(\Delta p)_{acoust.}}{1/2\ \rho_o u_o^2}$.

at any point, it can only be a function of the dimensionless parameters (i) to (vii) and of the dimensionless position of the point (x/L, y/L, z/L).

Moreover, if $\nu$ is any particular characteristic frequency of the noise, we can form a

(ix) "Dimensionless frequency": $F_o = \frac{\nu L}{u_o}$ ,

which is a function only of the dimensionless parameters.

In order to have similarity between two flow fields, we should have equality of the dimensionless parameters (i) to (vii). Thus, we must have geometrically similar flow boundaries and geometrically similar bodies in the flow field because of (i). In many flow problems of practical interest, the density variation of the fluid is small and can be approximated by assuming a constant compressibility*, considering the density a function of pressure only. This last assumption is justified either because temperature differences are small or because the thermal expansivity is small, or because temperature can be considered a function of pressure only, as in the isentropic expansion of a gas. Instead of introducing compressibility itself, it is more convenient to introduce the related parameter $a_o$, the velocity of sound in the fluid, and replace (ii) by

(x) Mach number, $Ma_o = \frac{u_o}{a_o}$ .

There are still a large number of parameters to be matched. However, if thermal effects are neglected, which seems permissible in many cases, (vi) and (vii) can be dropped. Moreover, if the fluid compressibility is small (as in a liquid), a change in $p_o$ has virtually no effect on the flow (except to change p everywhere by a constant amount) so that evidently $\pi_o$ and $G_o$ are not significant in this case.

It remains to match $Re_o$ and $Ma_o$. Such a matching for two similar flow fields of different scales is possible only if $\frac{\mu}{\rho_o a_o}$ is proportional

*Since we are interested in acoustic radiation, we cannot assume a zero compressibility for the fluid.

to the scale factor. This requires the use of two different liquids (or possibly the same liquid at two considerably different temperatures) which is experimentally inconvenient. If we wish to use the same liquid at about the same temperature, we can match only one of the two parameters $Re_o$ and $Ma_o$.

Consider first the flow around a body moving rectilinearly (i.e., exclude rotational motion as with a propeller) at low Mach number. Some noise will be produced by the formation and detachment of vortices on the surface of the body due to the action of viscosity.* For similarity of this noise field, two flow situations should have the same Reynolds number, and thus the velocities should be inversely proportional to the scale size L. From (viii) it follows that the acoustic pressure $(\Delta p)_{acoust.}$ at a distance L is inversely proportional to the square of the scale size. Hence, the total acoustic energy radiated per unit time, which is proportional to $(\Delta p)^2_{acoust.} \cdot 4\pi L^2$, varies with the inverse square of the scale factor (when the velocity is kept <u>inversely</u> proportional to the scale factor). In similar situations, $F_o$ will be constant; hence from (ix) the frequency spectrum of the noise will be shifted in this case by an amount inversely proportional to the square of the scale factor (i.e., if L is doubled, the noise frequency is lowered by two octaves). In nearly all flow situations of interest, the vortex noise is negligible, so that the relations given in the above paragraph have little practical significance.

In the more important case of propeller noise, the Mach numbers should be matched for similarity between two flow situations, while the effect of Reynolds number can usually be neglected. If the same fluid is used, equal velocities must be used, according to (x). Moreover, tangential velocities of the propeller at corresponding points (say, at

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*This corresponds to the "aerodynamic noise" of an airplane.

the propeller tip) must be kept equal so that the rpm of the propeller must vary inversely with the scale factor. With $u_o$ invariant, it follows from (viii) that $(\Delta p)_{acoust.}$ is invariant and hence the acoustic energy, which is proportional to $(\Delta p)^2_{acoust} \cdot 4\pi L^2$, varies with the square of the scale factor. It is seen from (ix) that the noise frequency must vary inversely with the scale factor (this also follows from the fact that the propeller rpm varies inversely with the scale factor).

We can use Gutin's equation[1] for propeller noise, which agrees well with experiments on airplane propellers, to test the relations derived above. Gutin's equations (10) and (13) can be combined to give

Acoustic power for $m^{\underline{th}}$ harmonic

$$= \int_0^{\pi} \frac{m^2 b^2 \omega^2}{4\pi \rho_o a_o^3} \left[ - \text{Thrust} \cdot \cos\theta + \frac{2a_o \cdot \text{Torque}}{b\,\omega\, R^2} \right]^2 J_{mb}(mb\omega R \sin\theta / a_o) \sin\theta \, d\theta$$

$$(m = 1, 2, 3, \ldots)$$

where b = number of blades of propeller, $\omega$ = angular frequency of rotation of propeller, R = the radius of the point of effective application of torque (approximately equal to 3/4 of the propeller radius), and $J_{mb}$ is the Bessel function of order mb. The integrand (without the $\sin\theta\, d\theta$) represents the intensity of noise radiated per unit solid angle at an angle $\theta$ with the direction toward which the propeller faces (thrust direction). The total acoustic output is determined by integration over $\theta$ and summation over all harmonics m.

Consider the scaling situation derived previously: $\rho_o$, $a_o$ and $u_o$ held fixed, L (and hence R) varied, and $\omega$ varied as 1/L. Evidently the term

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[1] Gutin, L. "On the Sound Field of a Rotating Propeller," NACA T.M. No. 1195 (1948), translated from Phys. Z. der Sowjetunion, 9, 57 (1936).

$\frac{m^2b^2\omega^2}{4\pi\rho_o a_o^3}$ will vary as $1/L^2$, while the argument ($mb\omega R \sin\theta/a_o$) of the Bessel function will be invariant since the variations of $\omega$ and R cancel. The velocity field, and hence the pressure field, about a given propeller blade element will be unchanged, so that the force is proportional to the area, whence the thrust varies with $L^2$ and the torque with $L^3$ (lever arm proportional to L). Both terms in the brackets then vary with $L^2$. Squaring this and multiplying by $1/L^2$ from the $\frac{m^2b^2\omega^2}{4\pi\rho_o a_o^3}$ term, one finds that the acoustic power radiated varies with $L^2$, which agrees with the result obtained from dimensional analysis.

## III. Hydrodynamic Noise in Cavitating Flow

For cavitating flow, the consideration of flow similarity is complicated by the appearance of new factors in the problem. These new factors arise since cavitation bubbles are initiated at nuclei in the liquid. These nuclei are presumably gas and vapor pockets stabilized on non-wetted impurities in the liquid. In ordinary water, there is most likely a large number of these nuclei. An analysis of scaling laws cannot be adequately made without a knowledge of the dynamics of the growth of cavitation bubbles from these nuclei. It is also desirable to have a knowledge of the dynamics of cavitation bubble collapse.

The theoretical studies made thus far indicate that the temperature changes associated with vapor bubble growth are not very great in the ordinary conditions of incipient cavitation.[2] If this is the case, one may neglect such quantities as specific heat, latent heat, and thermal diffusivity in the discussion of scaling laws. The parameters which remain to be considered are:

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[2] Plesset, M.S., "Dynamics of Cavitation Bubbles," Jour. of Applied Mechanics, vol. 16, pp. 277-283 (1949).

$\delta$ = an effective size for the nuclei in the liquid,

N = the number of nuclei per unit volume of the liquid,

$p_{v_o}$ = the vapor pressure of the liquid,

T = temperature of the liquid,

$p_{g_o}$ = the equilibrium pressure of dissolved gases,

$\sigma$ = surface tension of the liquid.

From these parameters the following dimensionless ratios may be formed:

(xi) Nucleus size ratio: $D = \delta/L$.

(xii) Surface tension number: $S = \sigma/Lp_o$.

(xiii) Dimensionless vapor pressure: $p_v = p_{v_o}/p_o$.

(xiv) Dimensionless equilibrium pressure of dissolved gases: $p_g = \dfrac{p_{g_o}}{p_o}$.

(xv) Nuclei number: $n = NL^3$.

With the neglect of thermal and heat diffusion factors, these dimensionless ratios, together with parameters (i), (iii), (iv), (v) and (x), are the quantities to be considered. The problem may be further simplified by the neglect of viscous and gravitational effects since, in many cavitation problems, they do not appear to be important. A discussion of cases where these effects are important will be given below.

A possible set of similarity conditions between two cavitating flows (1) and (2), with the neglect of the factors mentioned in the preceding paragraph, is the following:

(a) Geometric similarity with different scales $\underline{L_1}$ and $\underline{L_2}$.

(b) $p_o$, $u_o$, $\rho_o$, $a_o$, $p_{v_o}$, $p_{g_o}$, the same for the two flows; the equation of state for vapor and gases held the same.

(c) T the same for the two flows.

(d) Surface tension proportional to scale; i.e.,

$$\frac{\sigma_2}{\sigma_1} = \frac{L_2}{L_1}.$$

(e) Nucleus size proportional to scale, and nuclei number inversely proportional to the cube of the scale; i.e.,

$$\frac{\delta_2}{\delta_1} = \frac{L_2}{L_1}\,; \quad \frac{N_2}{N_1} = \frac{L_1^{\;3}}{L_2^{\;3}}\,.$$

The neglect of viscous, thermal and gravitational effects has reduced the modeling requirements to geometric similarity, to the use of the same liquid under the same temperature, pressure, velocity, density, vapor pressure, and concentration of dissolved gases. These requirements are easily satisfied experimentally.* The remaining requirements are that the surface tension be reduced as the scale is reduced, that the size of the nuclei be reduced with the scale, and that the number of the nuclei per unit volume be increased as the inverse third power of the scale. These latter requirements are difficult, if not impossible, to fulfill experimentally.

A complete analytic answer to the effects of surface tension, nucleus size and density on scale effects in cavitation is not available but some results of the analysis thus far completed may be given. It has been found possible to integrate the dynamic equation for the expansion of a nucleus into a cavitation bubble. This analysis shows that, if the effective initial size of the nuclei is sufficiently large so that surface tension effects are not important, the macroscopic bubble size is not a sensitive function of the initial size. Further, only a small fraction of the growth time is spent in the initial stages of bubble growth. Thus, the bubble history is not a sensitive function of the

*It should be noted that if these requirements are satisfied, the requirements for scaling of propeller noise without cavitation are also satisfied.

initial size. In this situation, the scaling requirement on the size of the nuclei would not be of great importance and similarity would obtain. The maximum size of the cavitation bubbles would be proportional to the scale factor. Furthermore, since any characteristic time, $\tau$ , has the form

$$\tau = \frac{L}{u_o} F(\pi_o, D, p_v, \ . \ . \ \text{etc.}),$$

the time of growth to maximum size would be proportional to the scale factor.

This conclusion, that the scale requirement on the size of the nuclei may be disregarded, must be qualified if small nuclei, for which surface tension effects are important, contribute appreciably to the cavitation in the flow.

From a practical point of view, one must suppose that the number of nuclei per unit volume is not changed as the scale is altered. Thus, for a model of reduced scale, there is a reduced number of nuclei available per unit reduced volume from which cavitation bubbles may grow. It is difficult to state quantitatively the effect of this deviation from the modeling requirement. One may consider two extreme possibilities:

Case 1. There is a large number of nuclei per unit volume which are effective centers for cavitation bubble formation, and a cavity is formed at only a small fraction of the available centers.

Case 2. There are only a few nuclei per unit volume of ordinary water which are effective centers for cavitation bubble formation, and a cavity is formed at each.

If Case 2 approximates the physical situation, then for a model of reduced scale the number of cavitation bubbles will not follow the similarity law, but will be reduced by the cube of the size reduction factor.

If Case 1 approximates the physical situation, then the number of cavitation bubbles will effectively be scaled. Let $\underline{L_2}$ be a characteristic size for the incipient cavitation flow situation to which one wishes to extrapolate information obtained from a geomentically similar flow situation with a reduced characteristic size $L_1$. The pressure, temperature, density, concentration of dissolved gases, and vapor pressure are taken to be the same for the two flows.

If the effective initial sizes of the nuclei for cavity formation are sufficiently large, then the cavitation bubbles in the large scale situation will grow to a maximum size which follows the scale factor $L_2/L_1$; the time of growth will be increased by the scale factor $L_2/L_1$; the time of collapse will also follow this same scale factor. In these respects, geometrical similarity between the two flows is preserved and the time scales follow the geometric factor as well.

Since $u_o$ is invariant, the acoustic pressure measured at a distance L is invariant, according to (viii), and hence the acoustic power which is proportional to $(\Delta p)^2_{acoust.} \cdot 4\pi L^2$, varies with the square of the scale factor. The sound frequency, according to (ix), varies inversely with the scale factor. These statements are valid only for Case 1 above, when there is an excess of cavity-forming nuclei in the liquid. In Case 2, when the number of nuclei is small, if the fairly reasonable assumption is made that the cavities do not interact, there will be an additional factor of $L^3$ in the scaling of the number of bubbles and the acoustic power will vary with the fifth power of the scale. The frequency variation of the noise will follow the inverse scale factor for both Case 1 and Case 2. This scale effect may be the explanation for some differences between laboratory and large scale observations on the frequency spectrum of cavitation noise. In the laboratory, the spectrum is flat quite far out. In large scale observations a drop in the intensity at the higher frequencies is observed. The present dimensional considerations predict a shift in the frequency spectrum by a factor 1/L which is in qualitative agreement with these observations.

If surface tension effects in the growth of bubbles from small nuclei are important, the general effect will be to increase the exponent in the scaling law for the cavitation-produced noise. This is seen from condition (d) above, which states that there would be modeling according to $L^2$ (Case 1) or $L^5$ (Case 2) provided that the surface tension is increased with the scale factor. If, more realistically, the surface tension is held constant instead of increased for the larger scale situation, the cavitation bubbles will evidently grow more rapidly and to a larger size, and the noise produced will increase faster than $L^2$ or $L^5$. This increase in the exponent may not be very great.

If the vertical dimension of the hydraulic body multiplied by $\rho_o g$ is not small compared to $p_o$, the hydrostatic pressure due to gravity will cause significantly more cavitation to occur on the top part than on the bottom part of the body. Hence (iv) must be matched by taking $p_o$ proportional to the scale factor, and hence $u_o$ proportional to the square root of the scale factor, from (ii). It follows that $Ma_o$ cannot be matched; however, its effect is probably unimportant in cavitation noise. From (xiii) and (xiv), the liquid vapor pressure and pressure of dissolved gases must be varied proportionately with the scale factor, while from (xii) the surface tension should be kept invariant. As before, we neglect the thermal, viscous, and nuclei size effects, and consider the two possible nuclei-number cases. This yields an acoustic power varying with the fourth power of the scale, for Case 1, or with the seventh power of the scale , for Case 2. From (ix) it is seen that the noise frequency is inversely proportional to the square root of the scale factor, for either case.

The present conclusions regarding the scaling laws for incipient cavitation noise do not apply directly to the situation in which the incipient cavitation is produced in vortices, such as are shed by propellers or as are formed on jet boundaries, for example. The formation of these vortices is a viscous effect and should, therefore, be affected by Reynolds number, while the modeling described above has not preserved Reynolds number. Since it is not convenient to attempt to alter the liquid viscosity or density appreciably, one may preserve Reynolds number by making the flow velocity $u_o$ inversely proportional to L. To preserve the pressure coefficient, one must alter the static pressure $p_o$ by the factor $1/L^2$. The vapor pressure, $p_{v_o}$, and dissolved gas pressure, $p_{g_o}$, must be altered similarly; this alteration requires a change in liquid temperature. In this similarity situation, the acoustic radiation varies with $1/L^2$; corresponding frequencies also vary with $1/L^2$.

Such a modeling procedure may not be required, since it seems reasonable to assume that a variation in Reynolds number will not have a strong effect on vortex behavior unless the flow is in some critical or unstable regime. The previous modeling which keeps velocities and pressures constant may give essentially the correct similarity conditions, for most cases of interest.

## IV. Summary of Scaling Laws

| Physical Situation | Velocity | Pressure | Vapor Pressure* | Propeller rpm | Noise Intensity | Noise Frequency |
|---|---|---|---|---|---|---|
| | Variation of Physical Parameters: Predetermined Parameters | | | | Resultant Parameters | |
| 1. Vortex noise without cavitation. | $L^{-1}$ | arbit. | arbit. | no prop. | $L^{-2}$ | $L^{-2}$ |
| 2. Propeller noise without cavitation. | const. | arbit. | arbit. | $L^{-1}$ | $L^{2}$ | $L^{-1}$ |
| 3. Cavitation noise, neglecting viscosity, gravity and surface tension; large number of nuclei (Case 1). | const. | const. | const. | $L^{-1}$ | $L^{2}$ | $L^{-1}$ |
| 4. Cavitation noise, neglecting viscosity, gravity and surface tension; limited number of nuclei (Case 2). | const. | const. | const. | $L^{-1}$ | $L^{5}$ | $L^{-1}$ |
| 5. Cavitation noise, surface tension important, neglecting viscosity and gravity; large number of nuclei (Case 1). | const. | const. | const. | $L^{-1}$ | $>L^{2}$ | ? |
| 6. Cavitation noise, surface tension important, neglecting viscosity and gravity; limited number of nuclei (Case 2). | const. | const. | const. | $L^{-1}$ | $>L^{5}$ | ? |
| 7. Cavitation noise, gravity important, neglecting viscosity and surface tension; large number of nuclei (Case 1). | $L^{1/2}$ | L | L | $L^{-1/2}$ | $L^{4}$ | $L^{-1/2}$ |
| 8. Cavitation noise, gravity important, neglecting viscosity and surface tension; limited number of nuclei (Case 2). | $L^{1/2}$ | L | L | $L^{-1/2}$ | $L^{7}$ | $L^{-1/2}$ |
| 9. Vortex cavitation noise, viscosity important, gravity and surface tension negligible. | $L^{-1}$ | $L^{-2}$ | $L^{-2}$ | $L^{-2}$ | $L^{-2}$ | $L^{-2}$ |

*and partial pressure of dissolved gases, if any.